Summer Learning 2016

Department: Math

Course Title: AP Calculus BC

Teacher: Ms. Lucic

ENTERING AP Calculus BC

To keep alive the techniques and knowledge you amassed in AP Calculus AB, your summer work will be to take two AP Calculus AB tests.

These tests are to completed and ready to discuss on the first day of class. Be sure to show all of your work for each problem. Problems will be graded for completeness, not correctness. Although you do not need to take them under "AP conditions", you may want to do so to accustom yourself to the time constraints imposed by the College Board.

Here's my suggestion: Try taking them under AP conditions and then go back and do the problems you could not do using your textbook as a resource.

If you get stuck and want to discuss something, please e-mail me at sarah.lucic@woosterschool.org.

Good luck!

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MATHEMATICS: CALCULUS AB

CALCULATORS AND REFERENCE MATERIALS MAY NOT BE USED IN THE EXAMINATION ROOM DURING THE TESTING PERIOD.

Three hours are allotted for this examination: 1 hour and 30 minutes for Section I, which consists of multiple-choice questions; and 1 hour and 30 minutes for Section II, which consists of longer problems. In determining your grade, the two sections are given equal weight. Section I is printed in this examination booklet; Section II, in a separate booklet.

SECTION I

Time—1 hour and 30 minutes

Number of questions—45

Percent of total grade—50

This examination contains 45 multiple-choice questions. Therefore, please be careful to fill in only the ovals that are preceded by numbers 1 through 45 on your answer sheet.

General Instructions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.

INDICATE ALL YOUR ANSWERS TO QUESTIONS IN SECTION I ON THE SEPARATE ANSWER SHEET ENCLOSED. No credit will be given for anything written in this examination booklet, but you may use the booklet for notes or scratchwork. After you have decided which of the suggested answers is best, COMPLETELY fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Example:

What is the arithmetic mean of the numbers 1, 3, and 6?

(A) 1 (B) $\frac{7}{3}$ (C) 3

(D) $\frac{10}{3}$ (E) $\frac{7}{2}$

Sample Answer

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. In this section of the examination, as a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. It is improbable, therefore, that mere guessing will improve your score significantly; it may even lower your score, and it does take time. If, however, you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices as wrong, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

Use your time effectively, working as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult. Go on to other questions and come back to the difficult ones later if you have time. It is not expected that everyone will be able to answer all the multiple-choice questions.

CALCULUS AB

SECTION I

Time—1 hour and 30 minutes

Number of questions—45

Percent of total grade—50

Directions: Solve each of the following problems, using the available space for scratchwork. Then decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in this examination booklet. Do not spend too much time on any one problem.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = x^2e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x(x+2e^x)$ (C) $xe^x(x+2)$

(D) $2x+e^x$ (E) $2x+e$

GO ON TO THE NEXT PAGE

2. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x: x \neq 3\}$ (B) $\{x: |x| \leq 2\}$ (C) $\{x: |x| \geq 2\}$

(D) $\{x: |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x: x \geq 2 \text{ and } x \neq 3\}$

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3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

GO ON TO THE NEXT PAGE

4. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x >$

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5. $\int \sec^2 x \, dx =$

(A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$

(D) $\frac{\sec^3 x}{3} + C$ (E) $2 \sec^2 x \tan x + C$

GO ON TO THE NEXT PAGE

6. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

---

7. $\int \frac{x\,dx}{\sqrt{3x^2 + 5}} =$

(A) $\frac{1}{9}(3x^2 + 5)^{\frac{3}{2}} + C$ (B) $\frac{1}{4}(3x^2 + 5)^{\frac{3}{2}} + C$ (C) $\frac{1}{12}(3x^2 + 5)^{\frac{1}{2}} + C$

(D) $\frac{1}{3}(3x^2 + 5)^{\frac{1}{2}} + C$ (E) $\frac{3}{2}(3x^2 + 5)^{\frac{1}{2}} + C$

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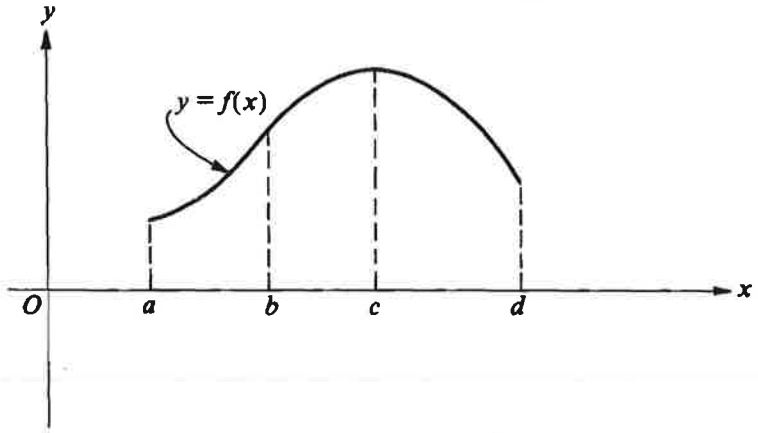

8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

GO ON TO THE NEXT PAGE

9. If $x + 2xy - y^2 = 2$, then at the point $(1, 1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

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10. If $\int_0^k (2kx - x^2)dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

GO ON TO THE NEXT PAGE

11. An equation of the line tangent to the graph of $f(x) = x(1 - 2x)^3$ at the point $(1, -1)$ is

(A) $y = -7x + 6$ (B) $y = -6x + 5$ (C) $y = -2x \cdot$

(D) $y = 2x - 3$ (E) $y = 7x - 8$

---

12. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

GO ON TO THE NEXT PAGE

13. If the function f has a continuous derivative on [0, c], then $\int_0^c f'(x)\,dx =$

(A) $f(c) - f(0)$ (B) $|f(c) - f(0)|$ (C) $f(c)$ (D) $f(x) + c$ (E) $f''(c) - f''(0)$

---

14. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1 + \sin\theta}}\,d\theta =$

(A) $-2(\sqrt{2} - 1)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$ (D) $2(\sqrt{2} - 1)$ (E) $2(\sqrt{2} + 1)$

GO ON TO THE NEXT PAGE

15. If $f(x) = \sqrt{2x}$, then $f'(2) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{}$

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16. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

GO ON TO THE NEXT PAGE

17. $\int_0^1 (3x - 2)^2 \, dx =$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

---

18. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

GO ON TO THE NEXT PAGE

19. $\int_2^3 \frac{x}{x^2+1}\,dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln$

---

20. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

GO ON TO THE NEXT PAGE

21. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

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22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

GO ON TO THE NEXT PAGE

23. If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

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24. $\frac{d}{dx}(x^{\ln x}) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)(x^{\ln x})$ (D) $(\ln x)(x^{\ln x - 1})$ (E) $2(\ln x)(x^{\ln x})$

GO ON TO THE NEXT PAGE

25. For all $x > 1$, if $f(x) = \int_1^x \frac{1}{t}\,dt$, then $f'(x) =$

(A) 1 (B) $\frac{1}{x}$ (C) $\ln x - 1$ (D) $\ln x$ (E) e^x

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26. $\int_0^{\frac{\pi}{2}} x \cos x\,dx =$

(A) $-\frac{\pi}{2}$ (B) -1 (C) $1 - \frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2} - 1$

GO ON TO THE NEXT PAGE

27. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geqq 3 \end{cases}$ is

(A) undefined

(B) continuous but not differentiable

(C) differentiable but not continuous

(D) neither continuous nor differentiable

(E) both continuous and differentiable

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28. $\int_1^4 |x - 3|\, dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

GO ON TO THE NEXT PAGE

29. The $\lim_{h \to 0} \dfrac{\tan 3(x + h) - \tan(3x)}{h}$ is

(A) 0 (B) $3 \sec^2 (3x)$ (C) $\sec^2 (3x)$ (D) $3 \cot (3x)$ (E) nonexistent

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30. A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the y-axis, the volume of the solid that is generated is represented by which of the following integrals?

(A) $2\pi \int_0^1 xe^{2x}\, dx$

(B) $2\pi \int_0^1 e^{2x}\, dx$

(C) $\pi \int_0^1 e^{4x}\, dx$

(D) $\pi \int_0^e y \ln y\, dy$

(E) $\frac{\pi}{4} \int_0^e \ln^2 y\, dy$

GO ON TO THE NEXT PAGE

31. If $f(x) = \frac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x) =$

(A) $\frac{x-1}{x}$ (B) $\frac{x+1}{x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x+1}$ (E) λ

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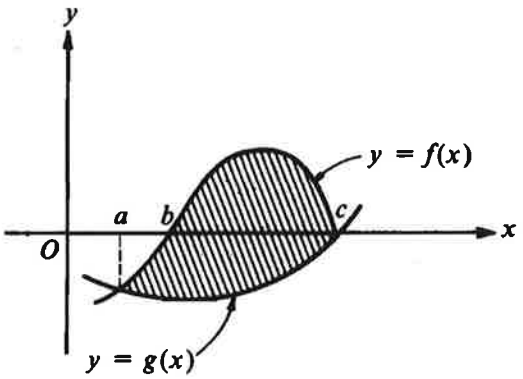
32. Which of the following does NOT have a period of π?

(A) $f(x) = \sin\left(\frac{1}{2}x\right)$ (B) $f(x) = |\sin x|$ (C) $f(x) = \sin^2 x$
(D) $f(x) = \tan x$ (E) $f(x) = \tan^2 x$

GO ON TO THE NEXT PAGE

33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2



34. The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)| - |g(x)|)dx$

(B) $\int_b^c f(x)dx - \int_a^c g(x)dx$

(C) $\int_a^c (g(x) - f(x))dx$

(D) $\int_a^c (f(x) - g(x))dx$

(E) $\int_a^b (g(x) - f(x))dx + \int_b^c (f(x) - g(x))dx$

GO ON TO THE NEXT PAGE

35. $4\cos\left(x + \frac{\pi}{3}\right) =$

(A) $2\sqrt{3}\cos x - 2\sin x$ (B) $2\cos x - 2\sqrt{3}\sin x$ (C) $2\cos x + 2\sqrt{3}$ si

(D) $2\sqrt{3}\cos x + 2\sin x$ (E) $4\cos x + 2$

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36. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

(A) -6 (B) -2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

GO ON TO THE NEXT PAGE

37. If $f(x) = e^x \sin x$, then the number of zeros of f on the closed interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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38. For $x > 0$, $\int \left(\frac{1}{x} \int_1^x \frac{du}{u} \right) dx =$

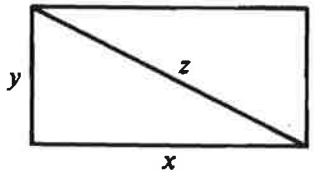
(A) $\frac{1}{x^3} + C$ (B) $\frac{8}{x^4} - \frac{2}{x^2} + C$ (C) $\ln(\ln x) + C$

(D) $\frac{\ln(x^2)}{2} + C$ (E) $\frac{(\ln x)^2}{2} + C$

GO ON TO THE NEXT PAGE

39. If $\int_1^{10} f(x)dx = 4$ and $\int_{10}^{3} f(x)dx = 7$, then $\int_1^3 f(x)dx =$

(A) -3 (B) 0 (C) 3 (D) 10 (E) 11



40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

GO ON TO THE NEXT PAGE

41. If $\lim_{x \to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

(A) None (B) II only (C) III only
(D) I and III only (E) I, II, and III

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42. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$ (B) $y = \sin x$ (C) $y = \frac{x}{x+1}$ (D) $y = \frac{x^2}{x-1}$ (E) $y = e^{-x}$

GO ON TO THE NEXT PAGE

43. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

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44. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only (B) II only (C) I and II only
(D) II and III only (E) I, II, and III

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MATHEMATICS : CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

It may be worthwhile for you to look through the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. The questions are printed in the booklet and on the green insert. It will be easier for you to look over all the problems on the insert; however, you should write all work for each problem in the space provided for that particular problem in the pink booklet. You may work the problems in any order. Do not spend too much time on any one problem.

Write all your answers in pencil only. Be sure to write CLEARLY and LEGIBLY. If you make an error, you may save time by crossing it out rather than trying to erase it.

Show all your work. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. All questions are given equal weight, but the parts of a particular question are not necessarily given equal weight. Credit for partial solutions will be given.

When you are told to begin, open your booklet, carefully tear out the green insert, and start work.

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

   (a) Find the domain of f.

   (b) Describe the symmetry, if any, of the graph of f.

Continue problem 1 on next page.

(c) Find $f'(x)$.

(d) Find the slope of the line normal to the graph of f at $x = 5$.

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2. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

(a) Find the acceleration $a(t)$ of the particle at any time t.

(b) Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.

Continue problem 2 on next page.

(c) Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

GO ON TO THE NEXT PAGE

3. Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$.

(a) Find the volume of the solid generated by revolving R about the x-axis.

Continue problem 3 on next page.

(b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the line $x = -1$.

GO ON TO THE NEXT PAGE

4. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

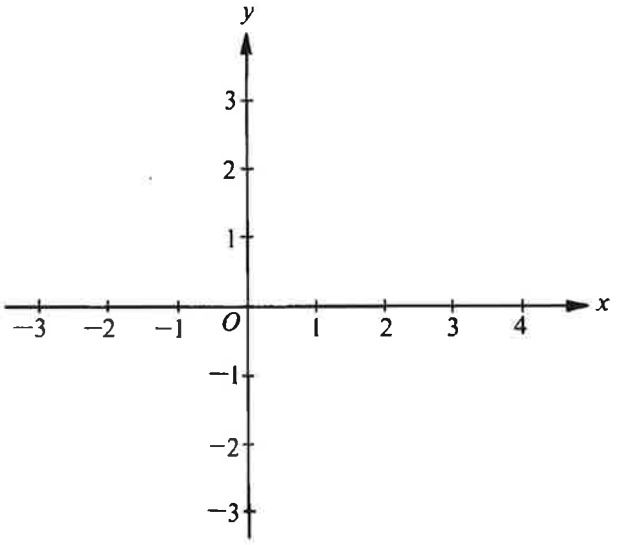
(a) Write an equation of the horizontal asymptote for the graph of f.

(b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.

Continue problem 4 on next page.

(c) For what values of x is the graph of f concave down?

(d) Using the results found in parts (a), (b), and (c), sketch the graph of $y = f(x)$ in the xy-plane provided below.



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5. Let R be the region in the first quadrant under the graph of $y = \frac{x}{x^2 + 2}$ for $0 \leq x \leq \sqrt{6}$.

(a) Find the area of R.

(b) If the line $x = k$ divides R into two regions of equal area, what is the value of k?

Continue problem 5 on next page.

(c) What is the average value of $y = \frac{x}{x^2 + 2}$ on the interval $0 \leqq x \leqq \sqrt{6}$?

GO ON TO THE NEXT PAGE

6. Let f be a differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$

(ii) $f'(1) = 6$ and $f''(1) = 18$

(iii) $\int_1^2 f(x)dx = 18$

Find $f(x)$. Show your work.

If more space is needed, continue problem 6 on next page.

CALCULUS AB

SECTION I, Part A

Time — 55 minutes

Number of questions — 28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10

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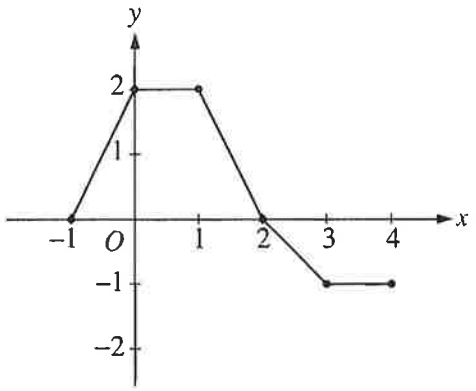

2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

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3. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2 \ln 2$

GO ON TO THE NEXT PAGE

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

(E) $\displaystyle\int_a^b f(x)\,dx$ exists.

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5. $\displaystyle\int_0^x \sin t\,dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

GO ON TO THE NEXT PAGE

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

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7. $\int_1^e \left(\frac{x^2 - 1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

GO ON TO THE NEXT PAGE

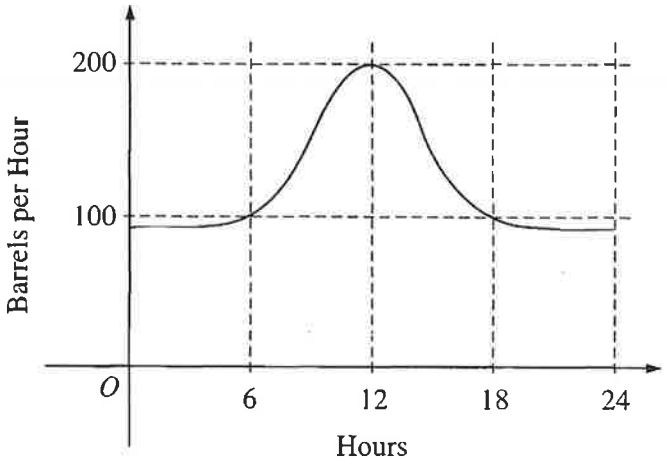
8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

GO ON TO THE NEXT PAGE

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

---

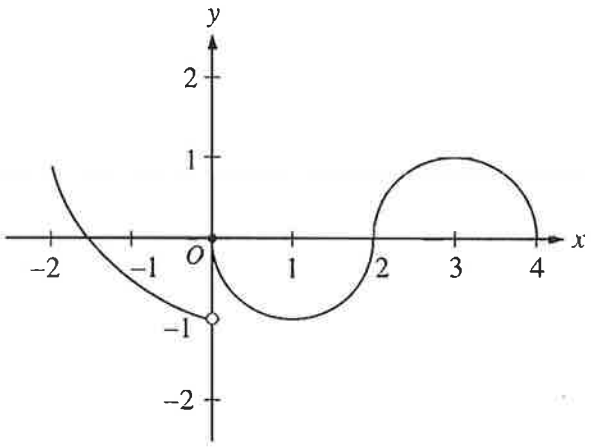
11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

GO ON TO THE NEXT PAGE

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, −1) and (3, 1). For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

GO ON TO THE NEXT PAGE

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

15. If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

GO ON TO THE NEXT PAGE

16. If $f(x) = \sin(e^{-x})$, then $f'(x) =$

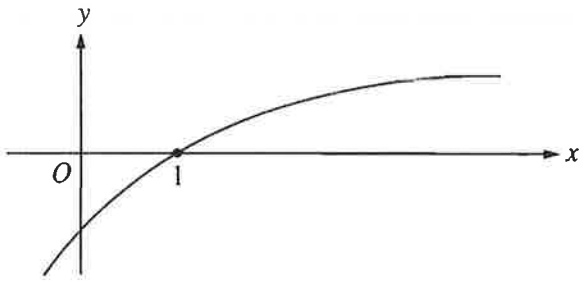
(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

GO ON TO THE NEXT PAGE

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0, 1)$ is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

---

19. If $f''(x) = x(x + 1)(x - 2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

---

GO ON TO THE NEXT PAGE

20. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$?

(A) -3 (B) 0 (C) 3 (D) -3 and 3 (E) -3, 0, and 3

---

21. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

GO ON TO THE NEXT PAGE

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

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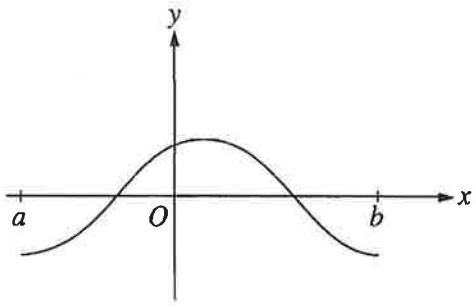
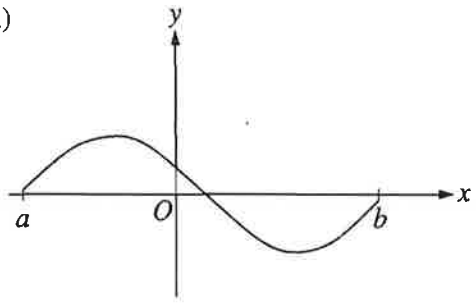
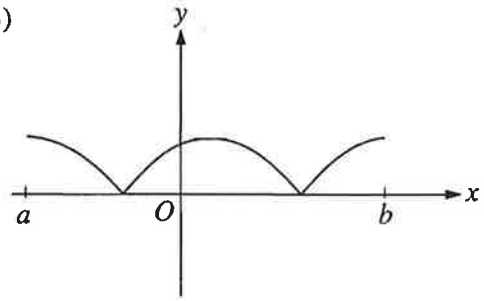


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

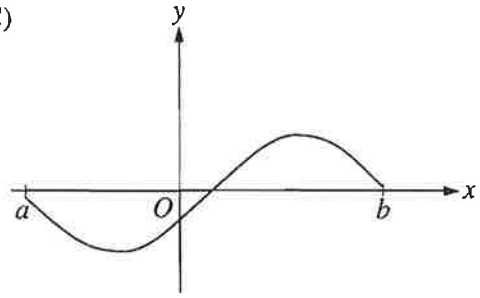
(A)



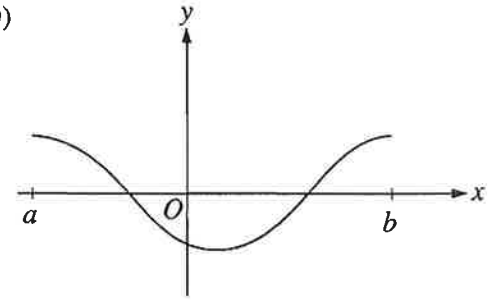
(B)



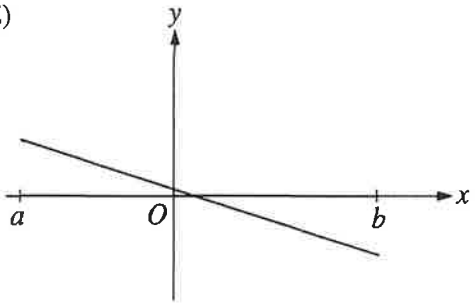
(C)



(D)



(E)



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24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

---

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

GO ON TO THE NEXT PAGE

x	0	1	2
$f(x)$	1	k	2

26. The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

---

27. What is the average value of $y = x^2\sqrt{x^3 + 1}$ on the interval [0, 2] ?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

GO ON TO THE NEXT PAGE

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS PART ONLY.
DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time — 50 minutes

Number of questions — 17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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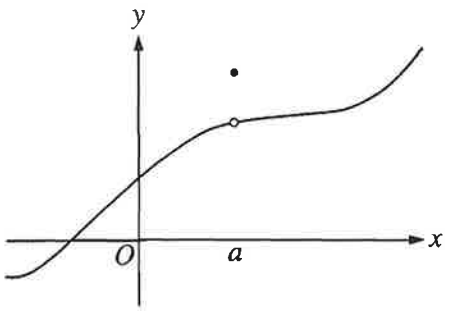

76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

GO ON TO THE NEXT PAGE

77. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) -0.701
(B) -0.567
(C) -0.391
(D) -0.302
(E) -0.258

---

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\dfrac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

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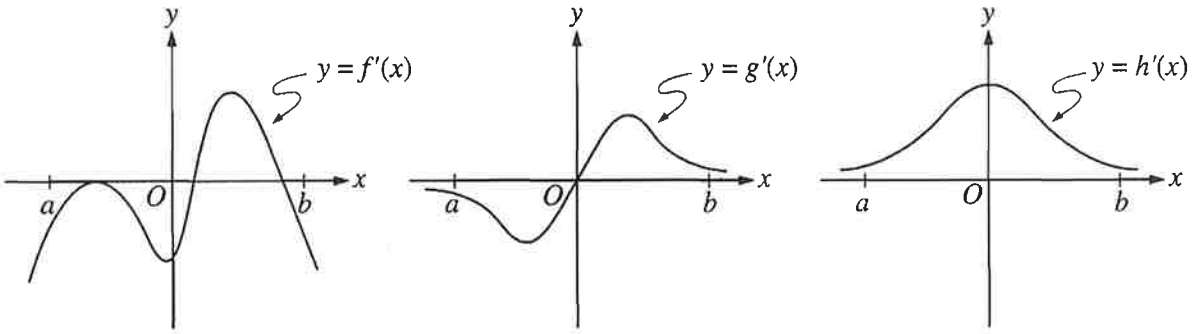


79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

---

80. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

GO ON TO THE NEXT PAGE

81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

---

82. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

GO ON TO THE NEXT PAGE

83. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

---

84. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

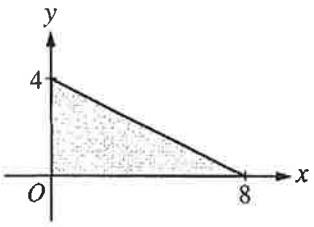
(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

GO ON TO THE NEXT PAGE

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210



86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

GO ON TO THE NEXT PAGE

87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

---

88. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

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89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

---

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

GO ON TO THE NEXT PAGE

91. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II and III

---

92. If $0 \leq k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART B ONLY.
DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET AND HAVE WRITTEN AND GRIDDED YOUR AP NUMBER IN THE APPROPRIATE SECTION OF YOUR ANSWER SHEET.

AFTER TIME HAS BEEN CALLED, ANSWER QUESTIONS 93-96.

93. Which graphing calculator did you use during the examination?

(A) Casio 6300, Casio 7000, Casio 7300, Casio 7400, or Casio 7700
(B) Texas Instruments TI-80 or TI-81
(C) Casio 9700, Casio 9800, Casio 9850, Sharp 9200, Sharp 9300, Texas Instruments TI-82, Texas Instruments TI-83, Texas Instruments TI-85, or Texas Instruments TI-86
(D) Hewlett-Packard HP-48 series or HP-38G
(E) Some other calculator

94. During your Calculus AB course, which of the following best describes your calculator use?

(A) I used my own graphing calculator.
(B) I used a graphing calculator furnished by my school, both in class and at home.
(C) I used a graphing calculator furnished by my school only in class.
(D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
(E) I did not use a graphing calculator.

95. During your Calculus AB course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?

(A) Almost every class
(B) About three-quarters of the classes
(C) About one-half of the classes
(D) About one-quarter of the classes
(E) Seldom or never

96. During your Calculus AB course, which of the following describes approximately how often you were allowed to use a graphing calculator on tests?

(A) Almost all of the time
(B) About three-quarters of the time
(C) About one-half of the time
(D) About one-quarter of the time
(E) Seldom or never

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight. The problems are printed in the booklet and in the green insert; it may be easier for you to first look over all problems in the insert. When you are told to begin, open your booklet, carefully tear out the green insert, and start to work.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

General instructions for this section are printed on the back cover of this booklet.

1. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

   (a) Find the area of the region R.

   (b) Find the value of h such that the vertical line $x = h$ divides the region R into two regions of equal area.

GO ON TO THE NEXT PAGE

(c) Find the volume of the solid generated when R is revolved about the x-axis.

(d) The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

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2. Let f be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

GO ON TO THE NEXT PAGE

(c) What is the range of f?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

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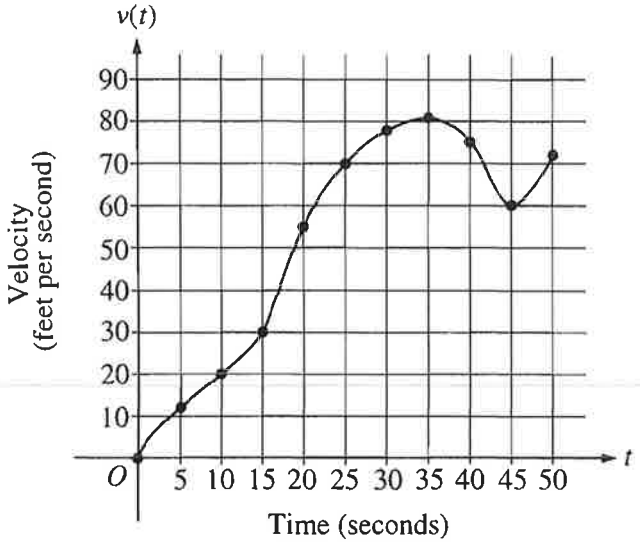


t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

GO ON TO THE NEXT PAGE

(c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

(d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

GO ON TO THE NEXT PAGE

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2 + 1}{2y}$.

(a) Find the slope of the graph of f at the point where $x = 1$.

(b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

GO ON TO THE NEXT PAGE

(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.

(d) Use your solution from part (c) to find $f(1.2)$.

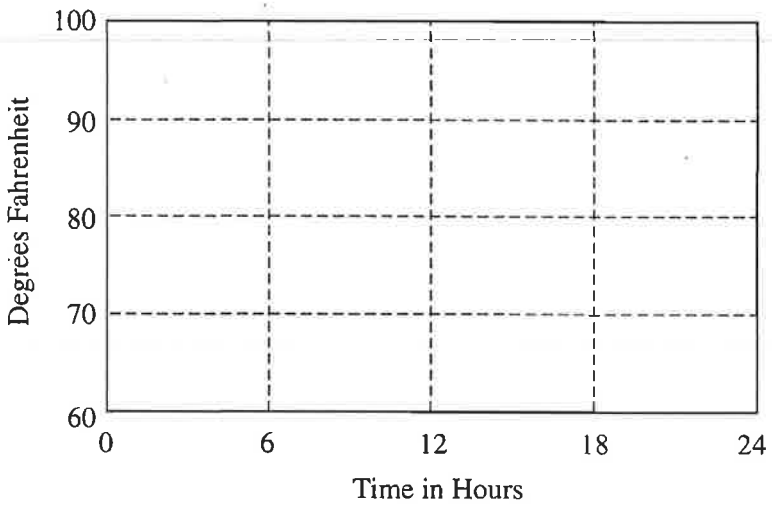
GO ON TO THE NEXT PAGE

5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right), \quad 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Sketch the graph of F on the grid below.



(b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

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(c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

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6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

(b) Write an equation of each horizontal tangent line to the curve.

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(c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x- and y-coordinates of point P.

END OF EXAMINATION

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